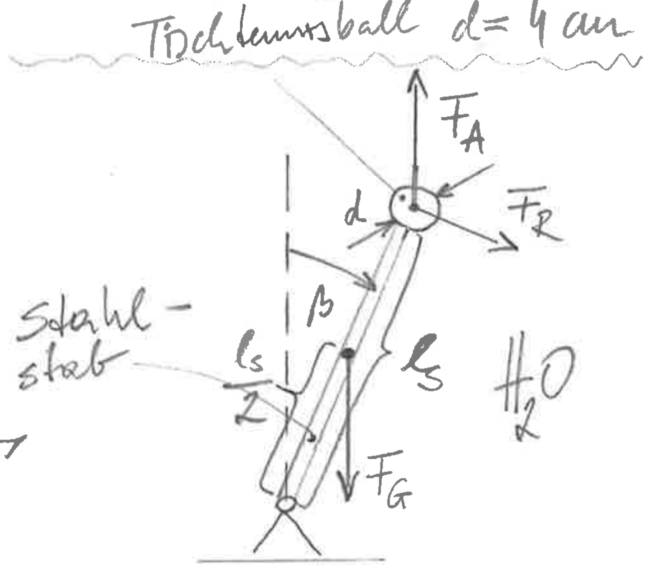
Inverses Pendel im Wasser



Annahmen:
1) $m_{Ball} \approx 0$
2) Reibungskraft des Stabes im Wasser ≈ 0
3) Auftriebskraft des Stabes ≈ 0

Lösg nur numerisch mögl.

$$-I\ddot{\beta} = -m_s \cdot g \cdot \frac{l}{2}\sin\beta + F_A \cdot \left(l + \overset{=r}{\frac{d}{2}}\right) \cdot \sin\beta - F_R \cdot \left(l + \frac{d}{2}\right)$$

$$-\frac{1}{3} m_s l^2 \ddot{\beta} = -m_s \cdot g \cdot \frac{l}{2}\underbrace{\sin\beta}_{\approx \beta,\ \text{wenn } \beta \downarrow\downarrow} + \rho_W \cdot V_B \cdot g\,(l+r)\underbrace{\sin\beta}_{\approx \beta,\ \text{wenn } \beta\downarrow\downarrow} - 6\pi\eta r\dot{\beta}(l+r)^2$$

analyt. (ohne Reibg.):

$$-I\ddot{\beta} = -m_s g\frac{l}{2}\beta + \rho_W V_B g\,(l+r)\,\beta$$

$$\ddot{\beta} = \frac{-m_s g\frac{l}{2} + \rho_W V_B g\,(l+r)}{-I}\,\beta$$

$$\ddot{\beta} - \frac{-m_s g\frac{l}{2} + \rho_W V_B g\,(l+r)}{-I}\,\beta = 0$$

$$\ddot{\beta} + \underbrace{\frac{\rho_W V_B g(l+r) - m_s g\frac{l}{2}}{I}}_{=\omega^2}\,\beta = 0$$

$r \ll l \Rightarrow l+r \approx l$

$$\omega = 2\pi f = \overset{+}{(-)}\sqrt{\frac{\rho_W V_B g(l+r) - m_s g\frac{l}{2}}{\underbrace{\frac{1}{3} m_s l^2}_{=I}}} \approx \sqrt{\frac{\rho_W V_B g - \frac{1}{2} m_s g}{\frac{1}{3} m_s l}}$$