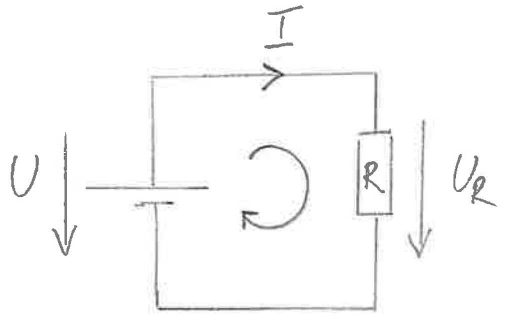
R, GS



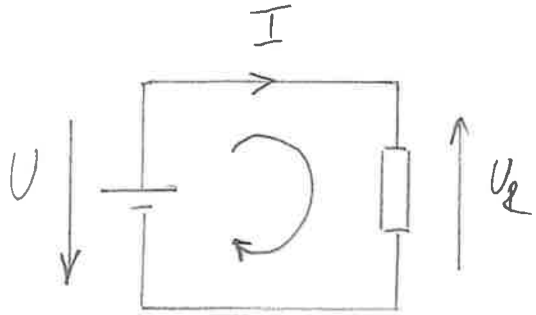
(VZS)

MR: $-U + U_R = 0$

$U = U_R$

$U = I \cdot R$

VZS (U_R & I zeigen in die gleiche Richtg)



(EZS)

MR: $-U - U_R = 0$

$U = -U_R$

$U = I \cdot R$

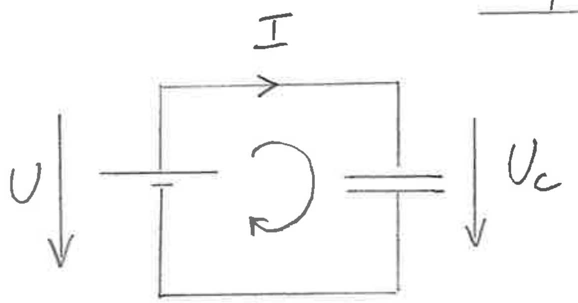
EZS (U_R & I zeigen in entgegengesetzte Richtungen)

⇒ Bauteilgesetze:

VZS : $U_R = I \cdot R$

EZS : $U_R = -I \cdot R$

C, GS

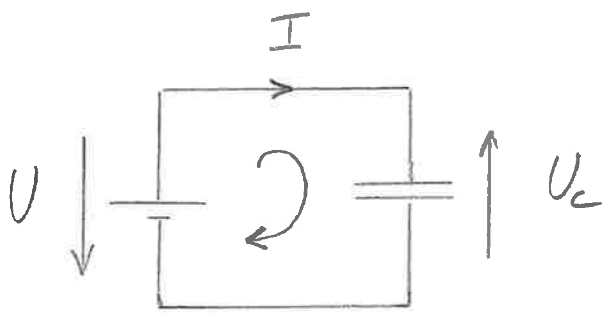


(VZS)

MR: $-U + U_c = 0$

$U = U_c$

$$I = \frac{dQ}{dt} = \frac{d}{dt}(C \cdot U_c) = C \cdot \underbrace{\frac{dU_c}{dt}}_{=0,\ \text{wenn } U_c = \text{const.}} = C \cdot \overbrace{\frac{dU}{dt}}^{=0,\ \text{wenn } U = \text{const.}} = 0$$



(EZS)

MR: $-U - U_c = 0$

$U = -U_c$

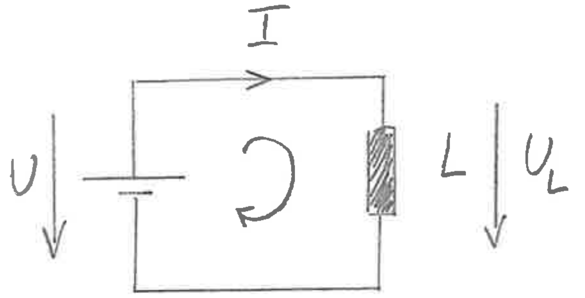
$$I = \frac{dQ}{dt} = \frac{d}{dt}(-C \cdot U_c) = -C \cdot \underbrace{\frac{dU_c}{dt}}_{=0,\ \text{s.o.}} = C \cdot \underbrace{\frac{dU}{dt}}_{=0,\ \text{s.o.}} = 0$$

$\Longrightarrow$ Bauteilgesetze:

VZS: $I = C \cdot \frac{dU_c}{dt}$

EZS: $I = -C \frac{dU_c}{dt}$

L, GS

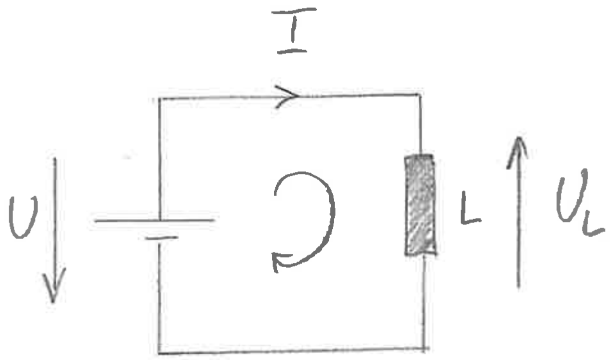


(VZS)

MR: $-U + U_L = 0$

$$U = U_L$$

$$U_L = L \cdot \frac{dI}{dt} = U$$



(EZS)

MR: $-U - U_L = 0$

$$U = -U_L$$

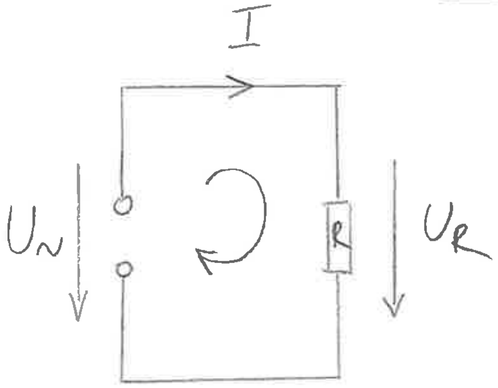
$$U_L = -L \cdot \frac{dI}{dt} = -U \Rightarrow U = L \frac{dI}{dt}$$

$\Rightarrow$ Bauteilgesetze:

VZS: $U_L = L \frac{dI}{dt}$

EZS: $U_L = -L \cdot \frac{dI}{dt}$

R, WS



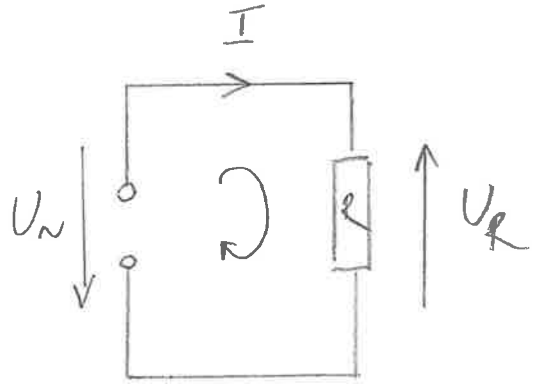
(VZS)

MR: $-U_\sim + U_R = 0$

$U_\sim = U_R$ mit $U_R = I \cdot R$

$U_0 \cdot \sin(\omega t) = I \cdot R$

$$I = \underbrace{\frac{U_0}{R}} \cdot \sin(\omega t) = I_0 \cdot \sin(\omega t)$$



(EZS)

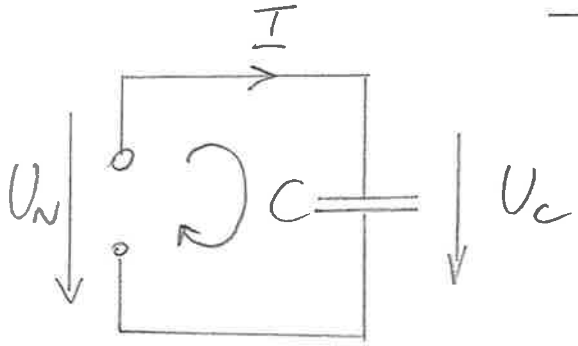
MR: $-U_\sim - U_R = 0$

$U_\sim = -U_R$ mit $U_R = -I \cdot R$

$U_0 \cdot \sin(\omega t) = I \cdot R$

$$I = \frac{U_0}{R} \cdot \sin(\omega t) = I_0 \cdot \sin(\omega t)$$

C, WS



(VZS)

$$\text{MR}: -U_N + U_C = 0$$

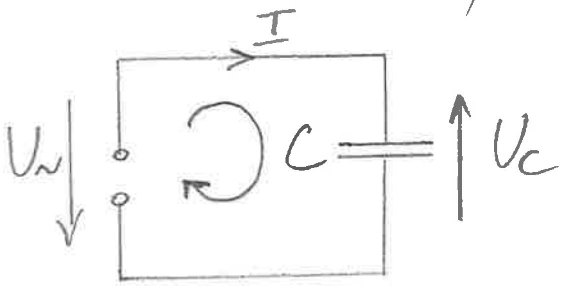
$$U_N = U_C \qquad \text{mit} \qquad I = C \cdot \frac{dU_C}{dt}$$

$$U_0 \cdot \sin(\omega t) = U_C$$

$$\Rightarrow I = C \cdot \frac{dU_C}{dt} = C \frac{d}{dt}\left(U_0 \cdot \sin(\omega t)\right) =$$

$$= C \cdot U_0 \frac{d}{dt}\left(\sin(\omega t)\right) = \underbrace{\omega C \cdot U_0}_{= I_0} \cdot \cos(\omega t) =$$

$$= I_0 \cdot \cos(\omega t)$$



(EZS)

$$\text{MR}: -U_N - U_C = 0$$

$$U_N = -U_C \qquad \text{mit} \qquad I = -C \cdot \frac{dU_C}{dt}$$

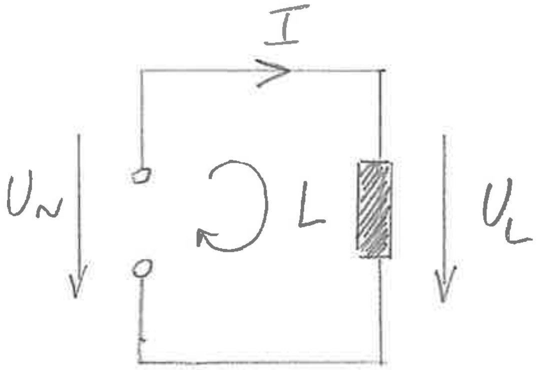
$$U_C = -U_C = -U_0 \cdot \sin(\omega t)$$

$$\Rightarrow I = -C \cdot \frac{dU_C}{dt} = -C \cdot \frac{d}{dt}\left(-U_0 \cdot \sin(\omega t)\right) =$$

$$= C U_0 \cdot \frac{d}{dt}\left(\sin(\omega t)\right) = \underbrace{\omega C \cdot U_0}_{= I_0} \cdot \cos(\omega t) =$$

$$= I_0 \cdot \cos(\omega t)$$

L, WS



(VZS)

MR: $-U_\sim + U_L = 0$

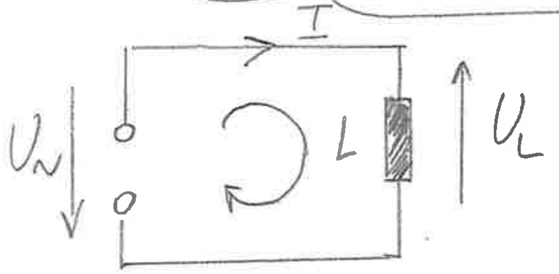
$U_\sim = U_L$ mit $U_L = L \cdot \frac{dI}{dt}$ & $U_\sim = U_0 \sin(\omega t)$

$U_L = U_0 \cdot \sin(\omega t)$

$\Rightarrow \quad dI = \frac{1}{L} \cdot U_L \cdot dt \Rightarrow I = \frac{1}{L} \cdot \int U_L \, dt$

$I = \frac{1}{L} \int U_0 \cdot \sin(\omega t) \, dt = \frac{1}{L} \cdot U_0 \int \sin(\omega t) \, dt =$

$= -\frac{1}{\omega L} \cdot U_0 \cdot \cos(\omega t) + I_{GS} = -I_0 \cdot \cos(\omega t) + I_{GS}$



(EZS)

MR: $-U_\sim - U_L = 0$

$U_\sim = -U_L$ mit $U_L = -L \frac{dI}{dt}$

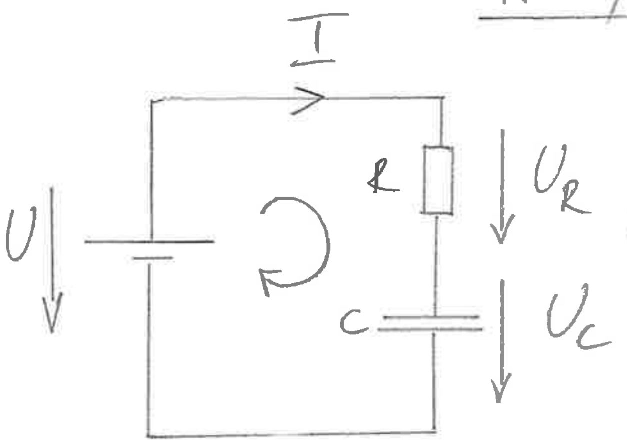
& $U_\sim = U_0 \cdot \sin(\omega t)$

$dI = -\frac{1}{L} \cdot U_L \, dt = -\frac{1}{L} (-U_\sim) \cdot dt =$

$= \frac{1}{L} U_\sim \, dt = \frac{1}{L} U_0 \cdot \sin(\omega t) \, dt$

$\Rightarrow I = \frac{1}{L} U_0 \cdot \int \sin(\omega t) \, dt = \underset{\dots}{(s.o.)} = -I_0 \cdot \cos(\omega t) + I_{GS}$

RC, GS, Ladung



(2x VtS)

MR: $U + U_R + U_C = 0$, $U = \text{const.}$

$$U = U_R + U_C \quad \text{mit} \quad U_R = I \cdot R \;\&\; \underbrace{I = C \cdot \frac{dU_C}{dt}}$$

$$\Downarrow$$

$$dU_C = \frac{I}{C} \cdot dt$$

$$U_C = \frac{1}{C}\int I\,dt$$

$$U = I \cdot R + \frac{1}{C}\int I\,dt$$

bzw. mit $I = \frac{dQ}{dt} \Leftrightarrow Q = \int I\,dt$

$$U = R \cdot \frac{dQ}{dt} + \frac{1}{C} Q$$

$$U - \frac{Q}{C} = R \cdot \frac{dQ}{dt}$$

$$dt = R \cdot \frac{dQ}{U - \frac{Q}{C}} \quad \Big| \int$$

$$\int dt = R \cdot \int \frac{dQ}{U - \frac{Q}{C}}$$

$$t + t_0 = R \cdot \ln\left(U - \frac{Q}{C}\right) \cdot (-C)$$

$$-\frac{1}{RC}(t + t_0) = \ln\left(U - \frac{Q}{C}\right) \quad \Big| e^{(\ldots)}$$

$$e^{-\frac{1}{RC}(t+t_0)} = U - \frac{Q}{C}$$

$$e^{-\frac{t}{RC}} \cdot e^{-\frac{t_0}{RC}} = U - \frac{Q}{C}$$

AB: $Q(t=0) = 0 \Rightarrow \underbrace{e^0}_{=1} \cdot e^{-\frac{t_0}{RC}} = U - \frac{0}{C}$

$$\Longrightarrow \quad e^{-\frac{t_0}{RC}} = U$$

$$\Longrightarrow e^{-\frac{t}{RC}} \cdot U = U - \frac{Q}{C}$$

$$\frac{Q}{C} = U - U \cdot e^{-\frac{t}{RC}}$$

$$Q = C \cdot U \cdot \left(1 - e^{-\frac{t}{RC}}\right)$$

$$U_c = \frac{Q}{C} = U\left(1 - e^{-\frac{t}{RC}}\right)$$

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(CU\left(1 - e^{-\frac{t}{RC}}\right)\right) =$$

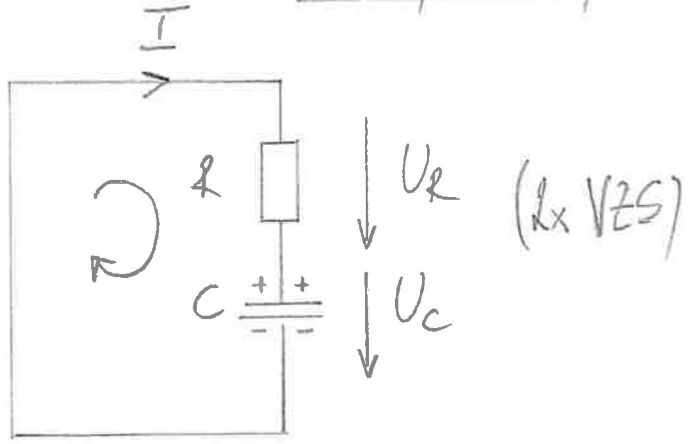
$$= C \cdot U \cdot \left(-e^{-\frac{t}{RC}}\right) \cdot \left(-\frac{1}{RC}\right) = \frac{\cancel{C} \cdot U}{R\cancel{C}} \cdot e^{-\frac{t}{RC}}$$

bzw.

$$I = C \cdot \frac{dU_c}{dt} = C \cdot \frac{d}{dt}\left(U \cdot \left(1 - e^{-\frac{t}{RC}}\right)\right) =$$

$$= \cancel{C} \cdot U \cdot \left(-e^{-\frac{t}{RC}}\right) \cdot \left(-\frac{1}{R\cancel{C}}\right) = \frac{U}{R} \cdot e^{-\frac{t}{RC}}$$

RC, GS, Entladung, Version 1



MR: $U_R + U_C = 0$

$$U_R = -U_C \quad \text{mit } U_R = I \cdot R \ \wedge \ I = C \cdot \frac{dU_c}{dt}$$

$$I \cdot R = -C \int I\, dt \qquad \text{bzw. } U_c = \frac{1}{C} \int I\, dt$$

$$\frac{dQ}{dt} R = -\frac{1}{C} \cdot Q$$

$$\frac{dQ}{Q} = -\frac{1}{RC}\, dt \quad \Big| \int$$

$$\ln Q = -\frac{1}{RC} \cdot t + k \quad \Big| e^{(\ldots)}$$

$$Q = e^{-\frac{t}{RC} + k} = e^{-\frac{t}{RC}} \cdot e^{k}$$

AB: $Q(t=0) = Q_0 \Rightarrow Q_0 = \underbrace{e^{0}}_{=1} \cdot e^{k}$

$$\Rightarrow Q = Q_0 \cdot e^{-\frac{t}{RC}}$$

$$U_C = \frac{Q}{C} = \underbrace{\frac{Q_0}{C}}_{= U_0 = U(t=0)} \cdot e^{-\frac{t}{RC}} = U_0 \cdot e^{-\frac{t}{RC}}$$

neg., da I-Richtung von − nach +

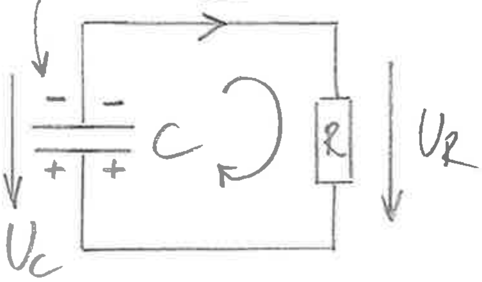
$$I = \frac{U_R}{R} = -\frac{U_C}{R} = -\frac{1}{R} \cdot U_0 \cdot e^{-\frac{t}{RC}} = - \overbrace{\frac{U_0}{R}}^{= I_0} \cdot e^{-\frac{t}{RC}}$$

$$I = \frac{dQ}{dt} = Q_0 \cdot \frac{d}{dt}\left(e^{-\frac{t}{RC}}\right) = -\frac{Q_0}{RC} \cdot e^{-\frac{t}{RC}} = -\frac{U_0}{R} \cdot e^{-\frac{t}{RC}}$$

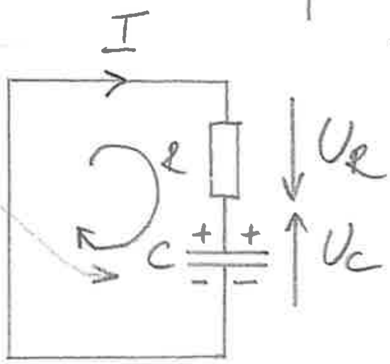
$=$

RC, GS, Entladung, Version 2

Begründung



(1x EZS, 1x VZS)



MR: $-U_C + U_R = 0$ mit $\underbrace{I = -C\,\frac{dU_C}{dt}}_{U_C = -\frac{1}{C}\cdot\underbrace{\int I\,dt}_{=Q}}$ & $U_R = I \cdot R$, $I = \frac{dQ}{dt}$

$$U_C = U_R$$

$$-\frac{Q}{C} = R \cdot \frac{dQ}{dt}$$

Lösung: siehe Version 1

$$Q = Q_0 \cdot e^{-\frac{t}{RC}}$$

$$U_C = -\frac{Q}{C} = -\underbrace{\frac{Q_0}{C}}_{=U_0} \cdot e^{-\frac{t}{RC}} = -U_0 \cdot e^{-\frac{t}{RC}}$$

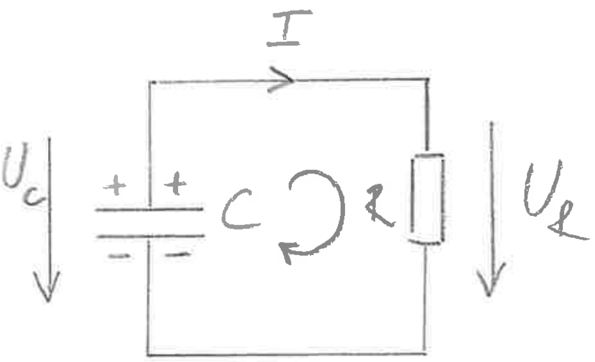
$$I = \frac{U_R}{R} = \frac{U_C}{R} = -\underbrace{\frac{U_0}{R}}_{=I_0} \cdot e^{-\frac{t}{RC}}$$

neg., da I-Richtung von − nach +

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(Q_0 \cdot e^{-\frac{t}{RC}}\right) = -\frac{Q_0}{RC}\, e^{-\frac{t}{RC}} =$$

$$= -\underbrace{\frac{U_0}{R}}_{=I_0} \cdot e^{-\frac{t}{RC}}$$

RC, GS, Entladung, Version 3



(1x ETS, 1x VZS)

MR: $-U_C + U_R = 0$ mit $\underbrace{I = -C \cdot \frac{dU_C}{dt}}$ $\wedge$ $U_R = I \cdot R$, $I = \frac{dQ}{dt}$

$$U_C = -\frac{1}{C}\underbrace{\int I\,dt}_{=Q}$$

$$U_C = -\frac{Q}{C}$$

$$U_C = U_R$$

$$-\frac{Q}{C} = R \cdot \frac{dQ}{dt}$$

Lösung: siehe Version 1

$$Q = Q_0 \cdot e^{-\frac{t}{RC}}$$

$$U_C = -\frac{Q_0}{C} \cdot e^{-\frac{t}{RC}} = -U_0 \cdot e^{-\frac{t}{RC}}$$

$$I = \frac{U_R}{R} = \frac{U_C}{R} = -\frac{U_0}{R} \cdot e^{-\frac{t}{RC}}$$

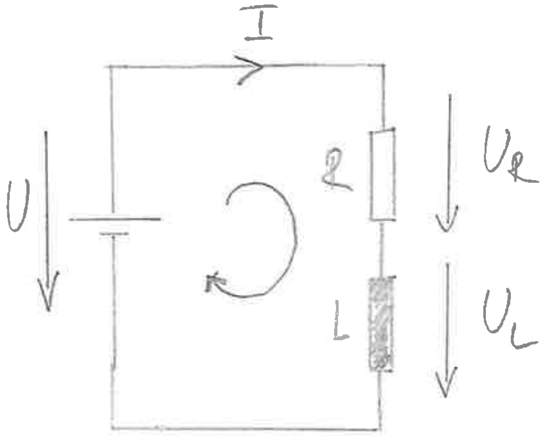
bzw./oder:

$$I = \frac{dQ}{dt} = Q_0 \cdot \frac{d}{dt}\left(e^{-\frac{t}{RC}}\right) = -\frac{Q_0}{RC}\, e^{-\frac{t}{RC}} =$$

$$= -\underbrace{\frac{U_0}{R}}_{=I_0} \cdot e^{-\frac{t}{RC}}$$

RL, GS, Magnetisierung

11/a



(2x VZS)

MR: $-U + U_R + U_L = 0$

$$U = U_R + U_L \quad \text{mit} \quad U_R = I \cdot R \;\&\; U_L = L\frac{dI}{dt}$$

$$U = I \cdot R + L\frac{dI}{dt}$$

$$L \cdot \frac{dI}{dt} = U - I \cdot R$$

$$\frac{dI}{U - I \cdot R} = \frac{1}{L}\, dt \quad \Big| \int$$

$$-\frac{1}{R} \cdot \ln(U - I \cdot R) = \frac{1}{L} \cdot t + k$$

$$\ln(U - I \cdot R) = -\frac{R}{L} \cdot t - Rk \quad \Big| e^{(\dots)}$$

$$U - I \cdot R = e^{-\frac{R}{L} \cdot t} \cdot e^{-Rk}$$

$$I \cdot R = U - e^{-\frac{R}{L} \cdot t} \cdot e^{-Rk}$$

$$I = \frac{U}{R} - \frac{1}{R} e^{-\frac{R}{L} \cdot t} \cdot e^{-Rk}$$

AB: $I(t=0) = 0 \Rightarrow \boxed{I =} \frac{U}{R} - \frac{1}{R} \cdot e^0 \cdot e^{-Rk} = 0$

$$\Rightarrow e^{-Rk} = U$$

$$\Rightarrow I = \underbrace{\frac{U}{R}}_{=I_\infty} - \underbrace{\frac{U}{R}}_{=I_\infty} \cdot e^{-\frac{R}{L} \cdot t} = I_\infty \cdot \left(1 - e^{-\frac{R}{L} \cdot t}\right)$$

$$U_L = L \cdot \frac{dI}{dt} = L \cdot \frac{d}{dt}\left(I_\infty\left(1-e^{-\frac{R}{L}t}\right)\right) =$$

$$= \cancel{L} \cdot I_\infty\left(-e^{-\frac{R}{L}t}\right) \cdot \left(-\frac{R}{\cancel{L}}\right) = +\underbrace{I_\infty \cdot R}_{=U} \cdot e^{-\frac{R}{L}t}$$

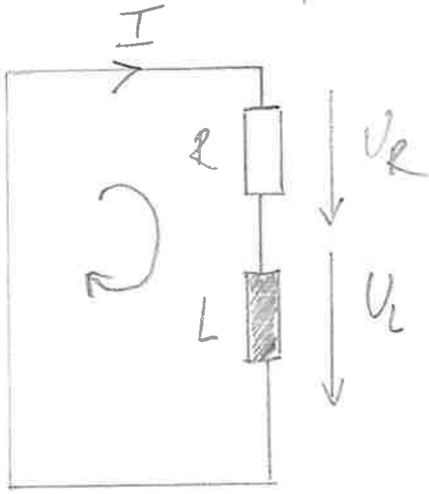
ODER:

$$U_L = U - U_R = U - I \cdot R = U - I_\infty \cdot \left(1-e^{-\frac{R}{L}t}\right) \cdot R =$$

$$= U - \underbrace{I_\infty \cdot R}_{=U} + I_\infty \cdot R \cdot e^{-\frac{R}{L} \cdot t} = \underbrace{I_\infty \cdot R}_{=U} \cdot e^{-\frac{R}{L}t}$$

$$U_L = U \cdot e^{-\frac{R}{L} \cdot t}$$

RL, GS, Entmagnetisierung



(2x VtS)

MR: $U_R + U_L = 0$

$U_R = -U_L$ mit $U_R = I \cdot R \wedge U_L = L\frac{dI}{dt}$

$I \cdot R = -L\frac{dI}{dt}$

$\frac{dI}{I} = -\frac{R}{L}\,dt \quad \Big| \int$

$\ln I = -\frac{R}{L} \cdot t + k \quad \Big| e^{(\ldots)}$

$I = e^{-\frac{R}{L}t} \cdot e^{k}$

AB: $I(t=0) = I_0 \Rightarrow I(t=0) = I_0 = \underbrace{e^{-0}}_{=1} \cdot e^{k}$

$\Rightarrow e^{k} = I_0$

$\Rightarrow I = I_0 \cdot e^{-\frac{R}{L}t}$

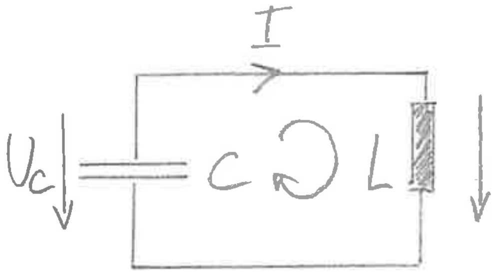
$U_L = L\frac{dI}{dt} = L \cdot \frac{d}{dt}\left(I_0 \cdot e^{-\frac{R}{L}t}\right) = \cancel{L} \cdot I_0 \cdot e^{-\frac{R}{L}t} \cdot \left(-\frac{R}{\cancel{L}}\right) =$

$= -I_0 \cdot R \cdot e^{-\frac{R}{L}t}$

ODER:

$U_L = -U_R = -I \cdot R = -I_0 \cdot e^{-\frac{R}{L} \cdot t} \cdot R = -I_0 R \cdot e^{-\frac{R}{L}t}$

LC, idealer Schwingkreis (frei)



(1x EZS, 1x VZS)

MR: $-U_C + U_L = 0$

$U_C = U_L$ mit $\underbrace{I = -C \cdot \frac{dU_C}{dt}}$ ∧ $U_L = L\frac{dI}{dt}$

$$U_C = -\frac{1}{C}\underbrace{\int I\,dt}_{=Q}, \quad \text{d.h. } I = \frac{dQ}{dt}$$

$$\Rightarrow -\frac{1}{C}\cdot Q = L\frac{dI}{dt}$$

$$-\frac{Q}{C} = L\frac{d^2Q}{dt^2}$$

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

<u>Ansatz:</u> $Q = Q_0 \cdot \cos(\omega t)$, wenn Kond. bei $t=0$ voll geladen

$$\frac{dQ}{dt} = \dot{Q} = -\omega Q_0 \sin(\omega t)$$

$$\frac{d^2Q}{dt^2} = \ddot{Q} = -\omega^2 Q_0 \cos(\omega t)$$

$$-\omega^2 \cancel{Q_0 \cos(\omega t)} + \frac{1}{LC}\cancel{Q_0 \cos(\omega t)} = 0$$

$$\Rightarrow \omega = \pm \frac{1}{\sqrt{LC}}$$

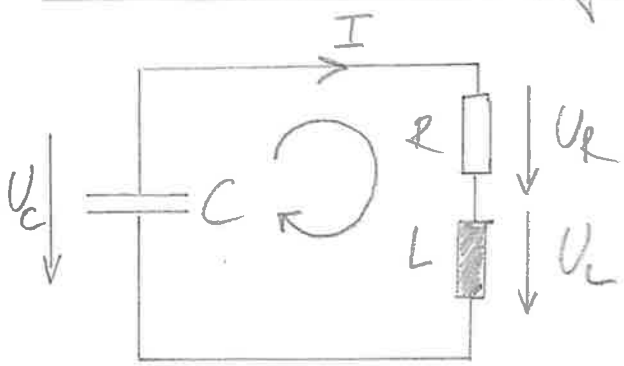
$$\to I = \frac{dQ}{dt} = -\underbrace{\omega Q_0}_{=I_0} \sin(\omega t)$$

$$U_C = -\frac{Q}{C} = -\underbrace{\frac{Q_0}{C}}_{=U_0} \cdot \cos(\omega t) = -U_0 \cdot \cos(\omega t)$$

$$U_L = L \cdot \frac{dI}{dt} = L \cdot \frac{d}{dt}\left(-\omega Q_0 \cdot \sin(\omega t)\right) = -\underbrace{L\omega^2}_{=\frac{1}{C}} Q_0 \cdot \cos(\omega t)$$

$\Leftrightarrow$ MR

realer LC-Schwingkreis „ohne Antrieb"



(1x EZS, 2x VZS)

MR: $-U_C + U_R + U_L = 0$

$$U_C = U_R + U_L \quad \text{mit} \quad U_C = -\frac{Q}{C} \text{ gemäß } I = -C\frac{dU_C}{dt}$$

$$U_R = I \cdot R$$

$$U_L = L\,\frac{dI}{dt}$$

$$-\frac{Q}{C} = I \cdot R + L\frac{dI}{dt} \quad \text{mit} \quad I = \frac{dQ}{dt}$$

$$\Rightarrow -\frac{Q}{C} = \frac{dQ}{dt}R + L\frac{d^2Q}{dt^2} \quad \Big| :(RL)$$

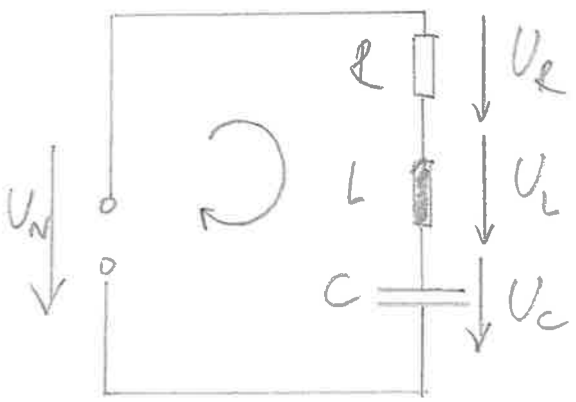
$$\frac{1}{R}\frac{d^2Q}{dt^2} + \frac{1}{L}\frac{dQ}{dt} + \frac{1}{RLC}Q = 0$$

Ansatz: $Q = Q_0 \cdot e^{-\delta t} \cdot \cos(\omega t)$ für voll geladenen Kond. bei $t=0$

Lösung siehe Maxima-Datei: RLC.WXMX

mit $U_0 = 0$

realer Schwingkreis mit Antrieb



3 x VZS

MR: $-U + U_R + U_L + U_C = 0$

$$U = U_R + U_L + U_C \quad \text{mit} \quad U_R = I \cdot R$$

$$U_L = L \frac{dI}{dt}$$

$$U_C = \frac{Q}{C}$$

$$\& \quad I = \frac{dQ}{dt}, \quad U_N = U_0 \sin(\omega t)$$

$$I \cdot R + L \frac{dI}{dt} + \frac{Q}{C} = U_0 \sin(\omega t)$$

$$L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} \cdot Q = U_0 \sin(\omega t)$$

Lösung siehe Maxima-Datei: RLC.wxmx